Practice Exam #2-C1- TOYS FOR PROFIT

TOYS N' THINGS, a manufacturing company, specializes in children's outdoor structures. Their newest products in development are the Hideout Hut™ and the Toy Castle™. The president of TOYS N' THINGS has hired you as a consultant. Your job is to choose only **one** product to sell in stores. You must choose the product that will make the most profit.

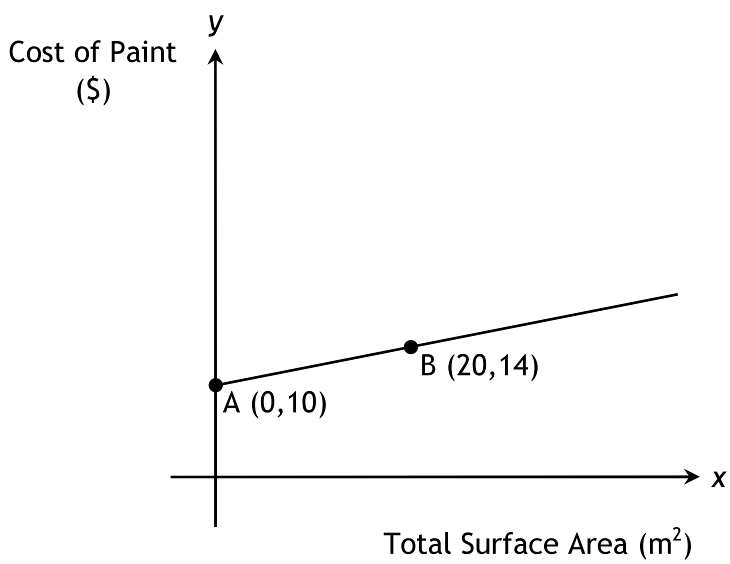
TOYS N' THINGS has set the retail price of both structures. The Hideout Hut sells for $99.95 and the Toy Castle sells for $73.95. The diagrams for both structures are shown on the next two pages of this booklet.

Cost of Material

Both structures are made of lightweight, solid plastic. The fixed cost for manufacturing the products is $7.50 plus an additional $6.75 per cubic metre of plastic used.

Cost of Painting

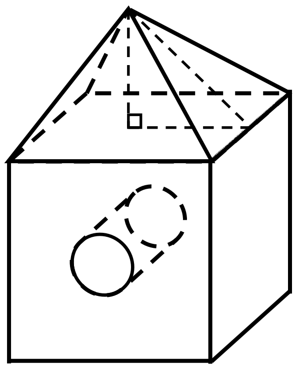
Both structures must also be painted. The cost of painting the structures is shown in the graph below.

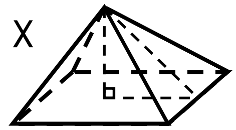


Which product would you recommend selling in order for TOYS N' THINGS to make the most profit?

Hideout Hut

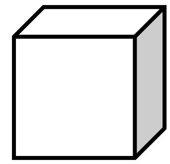
This is a pre-assembled structure with a cylindrical tunnel through the middle. The surface that touches the ground **does not** need to be painted. Only the exposed surfaces, including the inside of the tunnel, need to be painted. All of the surfaces are painted in the factory after assembly.



1 X 

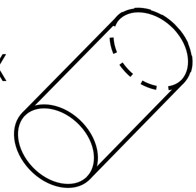
Square Based Pyramid

slant height: 1.5 m
base: 1.8 m

1 X 

Cube

side = 1.8 m

1 X 

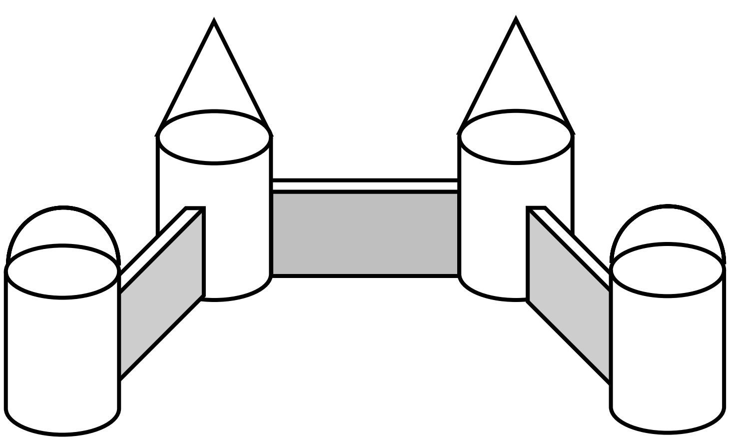
Cylinder

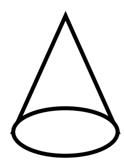
height: 1.8 m
choice of radius:
r = [0.2, 0.4] m

> **Note:**
>
> The cylinder radius can range from 0.2 m to 0.4 m. As a consultant you must **choose only one** size of tunnel (choose only one radius) when determining the costs.

Toy Castle

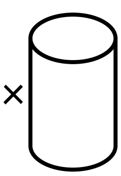
This structure will be assembled by the customer. All surfaces of the individual pieces are painted in the factory before assembly. The pieces used to create the structure are shown below.



2 

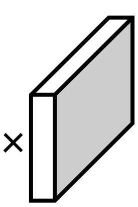
Cone

radius = 0.3 m
height = 0.4 m

4 × 

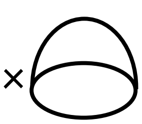
Cylinder

height: 1.5 m
radius: 0.3 m

3 × 

Rectangular Prism

length: 1.5 m
width: 0.1 m
height: 1.2

2 × 

Hemisphere

radius = 0.3 m

MARKING KEY OF THE SITUATIONAL PROBLEM

- **THE VOLUME OF THE HIDEOUT HUT**

Height of the pyramid

$$b = \frac{1.8}{2} = 0.9\ \mathbf{m}$$
$$h^2 = c^2 - b^2$$
$$= (1.5\ \mathbf{m})^2 - (0.9\ \mathbf{m})^2$$
$$= 1.44\ \mathbf{m}^2$$
$$\sqrt{h^2} = \sqrt{1.44\ \mathbf{m}^2}$$
$$h = 1.2\ \mathbf{m}$$

Volume of the pyramid

$$V = \frac{A_b h}{3}$$
$$= \frac{(s^2)h}{3}$$
$$= \frac{(1.8\text{ m})^2(1.2\text{ m})}{3}$$
$$= 1.30\text{ m}^3$$

Volume of the cube

$$V = s^3$$
$$= (1.8\ \mathbf{m})^3$$
$$= 5.83\ \mathbf{m}^3$$

Volume of cylinder

If r = 0.2 m

$$V = \pi r^2 h$$
$$= \pi(0.2\text{ m})^2(1.8\text{ m})$$
$$= 0.072\,\pi\text{ m}^2 \text{ or } 0.23\text{ m}^2$$

If r = 0.3 m

$$V = \pi r^2 h$$
$$= \pi(0.3\text{ m})^2(1.8\text{ m})$$
$$= 0.16\,\pi\text{ m}^2 \text{ or } 0.51\text{ m}^2$$

If r = 0.4 m

$$V = \pi r^2 h$$
$$= \pi(0.4\text{ m})^2(1.8\text{ m})$$
$$= 0.29\,\pi\text{ m}^2 \text{ or } 0.90\text{ m}^2$$

Total volume

If r = 0.2 m

$$V_T = V_{\text{cube}} + V_{\text{pyramid}} - V_{\text{cylinder}}$$
$$= 5.83\text{ m}^3 + 1.30\text{ m}^3 - 0.23\text{ m}^3$$
$$= 6.90\text{ m}^3$$

If r = 0.3 m

$$V_T = V_{\text{cube}} + V_{\text{pyramid}} - V_{\text{cylinder}}$$
$$= 5.83\text{ m}^3 + 1.3\text{ m}^3 - 0.51\text{ m}^3$$
$$= 6.62\text{ m}^3$$

If r = 0.4 m

$$V_T = V_{\text{cube}} + V_{\text{pyramid}} - V_{\text{cylinder}}$$
$$= 5.83\text{ m}^3 + 1.3\text{ m}^3 - 0.90\text{ m}^3$$
$$= 6.23\text{ m}^3$$

THE SURFACE AREA OF THE HIDEOUT HUT

Surface area of the pyramid

$$SA = 4A_{\text{lat}}$$
$$= 4\left(\frac{b \times h}{2}\right)$$
$$= 4\left(\frac{1.8\text{ m} \times 1.5\text{ m}}{2}\right)$$
$$= 5.4\text{ m}^2$$

Surface area of the cube (4 sides)

$$SA = 4s^2$$
$$= 4(1.8\text{ m})^2$$
$$= 12.96\text{ m}^2$$

Surface area of the inside of the tunnel (cylinder)

If r = 0.2 m

$$SA = A_{\text{lat}}$$
$$= 2\pi rh$$
$$= 2\pi(0.2\text{ m})(1.8\text{ m})$$
$$= 2.26\text{ m}^2 \text{ or } 0.72\pi\text{ m}$$

If r = 0.3 m

$$SA = A_{\text{lat}}$$
$$= 2\pi rh$$
$$= 2\pi(0.3\text{ m})(1.8\text{ m})$$
$$= 3.39\text{ m}^2 \text{ or } 1.08\,\pi\text{ m}$$

If r = 0.4 m

$$SA = A_{\text{lat}}$$
$$= 2\pi rh$$
$$= 2\pi(0.4\text{ m})(1.8\text{ m})$$
$$= 4.52\text{ m}^2 \text{ or } 1.44\text{ m}^2$$

Area of the circles

If r = 0.2 m

$$SA = 2\pi r^2 = 2\pi(0.2\text{ m})^2 = 0.25\text{ m}^2 \quad \text{or} \quad 0.08\pi\text{ m}^2$$

If r = 0.3 m

$$SA = 2\pi r^2 = 2\pi(0.3\text{ m})^2 = 0.57\text{ m}^2 \quad \text{or} \quad 0.18\pi\text{ m}^2$$

If r = 0.4 m

$$SA = 2\pi r^2 = 2\pi(0.4\text{ m})^2 = 1.00\text{ m}^2 \quad \text{or} \quad 0.32\pi\text{ m}^2$$

Total surface area

If r = 0.2 m

$$\begin{aligned} TSA &= A_{\text{cube}} + A_{\text{pyramid}} + A_{\text{Lat, cylinder}} - A_{\text{circles}} \\ &= 12.96\text{ m}^2 + 5.4\text{ m}^2 + 2.26\text{ m}^2 - 0.25\text{ m}^2 \\ &= 20.37\text{ m}^2 \end{aligned}$$

If r = 0.3 m

$$\begin{aligned} TSA &= A_{\text{cube}} + A_{\text{pyramid}} + A_{\text{Lat, cylinder}} - A_{\text{circles}} \\ &= 12.96\text{ m}^2 + 5.4\text{ m}^2 + 3.39\text{ m}^2 - 0.57\text{ m}^2 \\ &= 21.18\text{ m}^2 \end{aligned}$$

If r = 0.4

$$\begin{aligned} TSA &= A_{\text{cube}} + A_{\text{pyramid}} + A_{\text{Lat, cylinder}} - A_{\text{circles}} \\ &= 12.96\text{ m}^2 + 5.4\text{ m}^2 + 4.52\text{ m}^2 - 1.00\text{ m}^2 \\ &= 21.88\text{ m}^2 \end{aligned}$$

- **THE VOLUME OF THE TOY CASTLE**

Volumes of cones

$$\begin{aligned} V_{\text{cone}} &= \frac{\pi r^2 h}{3} \\ &= \frac{\pi(0.3)^2(0.4)}{3} \\ &= 0.038\ \mathbf{m}^3 \end{aligned}$$

$$\begin{aligned} 2\times V_{\text{cone}} &= 2\times 0.038\ \mathbf{m}^3 \\ &= 0.08\ \mathbf{m}^3 \end{aligned}$$

Volume of the cylinders

$$\begin{aligned} V_{\text{cylinder}} &= \pi r^2 h \\ &= \pi(0.3\ \mathbf{m})^2(1.5\ \mathbf{m}) \\ &= 0.135\pi\ \mathbf{m}^3 \\ &\ \ \textbf{or}\quad 0.424\ \mathbf{m}^3 \\ 4\times V_{\text{cylinder}} &= 4\times 0.42\ \mathbf{m}^3 \\ &= 1.70\ \mathbf{m}^3 \end{aligned}$$

Volumes of rectangular prisms

$$\begin{aligned} V_{\text{prism}} &= l\times w\times h \\ &= (1.2\ \mathbf{m})(0.1\ \mathbf{m})(1.5\ \mathbf{m}) \\ &= 0.18\ \mathbf{m}^3 \end{aligned}$$

$$\begin{aligned} 3\times V_{\text{prism}} &= 3\times 0.18\ \mathbf{m}^3 \\ &= 0.54\ \mathbf{m}^3 \end{aligned}$$

Volumes of hemispheres

$$\begin{aligned} V_{\text{hemisphere}} &= \frac{4\pi r^3}{6} \\ &= \frac{4\pi(0.3)^3}{6} \\ &= 0.057\text{ m}^3 \end{aligned}$$

$$\begin{aligned} 2\times V_{\text{hemisphere}} &= 2\times 0.057\text{ m}^3 \\ &= 0.11\text{ m}^3 \end{aligned}$$

Total volume

$$\begin{aligned} V_{\text{total}} &= V_{\text{cylinders}} + V_{\text{cones}} + V_{\text{hemisphere s}} + V_{\text{prisms}} \\ &= 1.70\ \mathbf{m}^3 + 0.08\ \mathbf{m}^3 + 0.11\ \mathbf{m}^3 + 0.54\ \mathbf{m}^3 \\ &= 2.43\ \mathbf{m}^3 \end{aligned}$$

- **THE SURFACE AREA OF THE TOY CASTLE**

Slant Height of the cone

$$\begin{aligned} SL^2 &= r^2 + h^2 \\ &= (0.3\text{ m})^2 + (0.4\text{ m})^2 \\ SL^2 &= 0.25\text{ m}^2 \\ \sqrt{SL^2} &= \sqrt{0.25\text{ m}^2} \\ SL &= 0.5\text{ m} \end{aligned}$$

Surface area of the cones

$$\begin{aligned} SA_{\text{cone}} &= A_{\text{Lat}} + A_{\text{base}} \\ &= \pi r SL + \pi r^2 \\ &= \pi(0.3\text{ m})(0.5\text{ m}) + \pi(0.3)^2 \\ &= 0.75\text{ m}^2 \quad \text{or} \quad 0.24\pi\text{ m}^2 \end{aligned}$$

$$\begin{aligned} 2\times SA_{\text{cone}} &= 2\times 0.75\text{ m}^2 \\ &= 1.50\text{ m}^2 \end{aligned}$$

Surface area of the cylinder

$$\begin{aligned} SA_{\text{cylinder}} &= A_{\text{Lat}} + 2A_{\text{base}} \\ &= 2\pi rh + 2\pi r^2 \\ &= 2\pi(0.3\ \mathbf{m})(1.5\ \mathbf{m}) + 2\pi(0.3)^2 \\ &= 3.\mathbf{39}\ \mathbf{m}^2 \quad \textbf{or} \quad \mathbf{0.36}\pi\ \mathbf{m}^2 \end{aligned}$$

$$\begin{aligned} 4\times SA_{\text{cylinder}} &= 4\times \mathbf{3.39}\ \mathbf{m}^2 \\ &= \mathbf{13.56}\ \mathbf{m}^2 \end{aligned}$$

Surface area of the rectangular prisms

$$
\begin{aligned}
SA_{\text{prism}} &= A_{\text{base}} + A_{\text{Lat}} \\
&= 2(l \times w) + P_{\text{base}}\, h \\
&= \mathbf{2(1.5\ m)(0.1\ m) + (2 \times 1.5\ m + 2 \times 0.1\ m)(1.2\ m)} \\
&= 4.14\ \mathbf{m}^2
\end{aligned}
$$

$$
\begin{aligned}
3 \times SA_{\text{prism}} &= \mathbf{3 \times 4.14\ m^2} \\
&= \mathbf{12.42\ m^2}
\end{aligned}
$$

Surface area of the hemispheres

$$
\begin{aligned}
SA_{\text{hemistere}} &= A_{\text{Lat}} + A_{\text{Base}} \\
&= \frac{4\pi r^2}{2} + \pi r^2 \\
&= 2\pi(0.3\ \text{m})^2 + \pi(0.3)^2 \\
&= 0.85\ \text{m}^2 \ \text{ or } 0.36\pi\ \text{m}^2
\end{aligned}
$$

$$
\begin{aligned}
2 \times SA_{\text{hemisphere}} &= 2 \times 0.85\ \text{m}^2 \\
&= 1.7\ \text{m}^2
\end{aligned}
$$

Total surface area

$$
\begin{aligned}
TSA &= SA_{\text{cylinders}} + SA_{\text{cones}} + SA_{\text{hemisphere s}} + SA_{\text{prisms}} \\
&= 13.56\ \text{m}^2 + 1.5\ \text{m}^2 + 1.7\ \text{m}^2 + 12.42\ \text{m}^2 \\
&= 29.18\ \text{m}^2
\end{aligned}
$$

- **THE RULE FOR THE COST ($) vs VOLUME OF MATERIAL (m^3)**

$$
\begin{aligned}
y &= ax + b \\
y &= 6.75x + 7.50
\end{aligned}
$$

- **THE RULE FOR THE COST ($) OF PAINTING ($m^2$)**
  Find the slope given two points A(0,10) and B(10,14)

$$
\begin{aligned}
a &= \frac{y_2 - y_1}{x_2 - x_1} \\
a &= \frac{(14 - 10)}{(10 - 0)} \\
a &= \frac{4}{10} \\
a &= 0.2
\end{aligned}
$$

Therefore $y = 0.2x + 10$

- **THE TOTAL COST OF THE HIDEOUT HUT**

Cost of the Hideout Hut Materials (m^3)

If r = 0.2 m

$$
\begin{aligned}
y &= 6.75x + 7.50 \\
y &= 6.75(6.90) + 7.50 \\
y &= \$54.08
\end{aligned}
$$

If r = 0.3 m

$$
\begin{aligned}
y &= 6.75x + 7.50 \\
y &= 6.75(6.62) + 7.50 \\
y &= \$52.19
\end{aligned}
$$

If r = 0.4 m

$$
\begin{aligned}
y &= 6.75x + 7.50 \\
y &= 6.75(6.23) + 7.50 \\
y &= \$49.55
\end{aligned}
$$

Cost of Painting the Hideout Hut (m^2)

If r = 0.2 m

$$
\begin{aligned}
y &= 0.2x + 10 \\
y &= 0.2(20.37) + 10 \\
y &= \$14.07
\end{aligned}
$$

If r = 0.3 m

$$
\begin{aligned}
y &= 0.2(21.18) + 10 \\
y &= \$14.24
\end{aligned}
$$

If r = 0.4 m

$$
\begin{aligned}
y &= 0.2(21.88) \\
y &= \$14.38
\end{aligned}
$$

Total cost of the Hideout Hut

If r = 0.2 m

$$
\begin{aligned}
\text{Cost} &= \text{Materials} + \text{Paint} \\
&= 54.08 + 14.07 \\
&= \$68.15
\end{aligned}
$$

If r = 0.3 m

$$
\begin{aligned}
&= \text{Materials} + \text{Paint} \\
&= 52.19 + 14.24 \\
&= \$66.43
\end{aligned}
$$

If r = 0.4 m

$$
\begin{aligned}
&= \text{Material} + \text{Paint} \\
&= 49.55 + 14.38 \\
&= \$63.93
\end{aligned}
$$

- **THE PROFIT FOR THE HIDEOUT HUT**

If r = 0.2 m

$$
\begin{aligned}
\textit{Profit} &= \textit{Sale Price} - \textit{Cost} \\
&= 99.95 - 68.15 \\
&= \$31.80
\end{aligned}
$$

If r = 0.3 m

$$
\begin{aligned}
&= \textit{Sale Price} - \textit{Cost} \\
&= 99.95 - 66.43 \\
&= \$33.52
\end{aligned}
$$

If r = 0.4 m

$$
\begin{aligned}
&= \textit{Sale Price} - \textit{Cost} \\
&= 99.95 - 63.93 \\
&= \$36.02
\end{aligned}
$$

- **THE TOTAL COST OF THE TOY CASTLE**

Cost of the Toy Castle Material (m^3)

$y = 6.75x + 7.50$

$y = (6.75)(2.43) + 7.50$

$y = \$23.90$

Cost of painting the toy castle (m^2)

$y = 0.2x + 10$

$y = (0.2)(29.18) + 10$

$y = \$15.84$

- **Total Cost of the Toy Castle**

$\text{Cost} = \text{Materials} + \text{Paint}$

$= 23.90 + 15.84$

$= \$39.74$

THE PROFIT FOR THE TOY CASTLE

$\text{Profit} = \text{Sale Price} - \text{Cost}$

$= 73.95 - 39.74$

$= \$34.21$

- **Possible Answers**

Option 1 (cylinder with radius = 0.2 m)

I would recommend that TOYS N' THINGS sell the **Toy Castle™** ($34.21) over the Hideout Hut™ ($31.80) since it generates a greater profit (**$2.41**).

Option 2 (cylinder with radius = 0.3 m)

I would recommend that TOYS N' THINGS sell the **Toy Castle™** ($34.21) over the Hideout Hut™ ($33.52) since it generates a greater profit ($0.69).

Option 3 (cylinder with radius = 0.4 m)

I would recommend that TOYS N' THINGS sell the **Hideout Hut™** ($36.02) instead of the Toy Castle™ ($34.21) since it generates a greater profit ($1.81).

***Option 4** (cylinder with radius = 0.33 m)

I would recommend that TOYS N' THINGS sell ***EITHER*** the Hideout Hut™ ***OR*** the Toy Castle™ since they both generate the same amount of profit ($34.21).

Hideout Hut	Total Volume	Total Area	Cost Volume	Cost Area	Total Cost	Selling Price	Profit
Cylinder with radius of 0.2 m	6.90 m^3	20.37 m^2	$54.08	$14.07	$68.15	$99.95	**$31.80**
Cylinder with radius of 0.3 m	6.62 m^3	21.18 m^2	$52.19	$14.24	$66.43	$99.95	**$33.52**
Cylinder with radius of 0.4 m	6.23 m^3	21.88 m^2	$49.55	$14.38	$63.93	$99.95	**$36.02**
Cylinder with radius of 0.33 m	6.51 m^3	21.41 m^2	$51.47	$14.27	$65.74	$99.95	**$34.21**

Toy Castle	Total Volume	Total Area	Cost Volume	Cost Area	Total Cost	Selling Price	Profit
Toy Castle	2.43 m^3	29.18 m^2	$23.90	$15.84	$39.74	$73.95	**$34.21**

*The **Toy Castle™** will generate more profit for any radii $\in [0.2, 0.33[$ metres.

*The **Hideout Hut™** will generate more profit for any radii $\in \,]0.33, 0.4]$ metres.