

# Practice Exam-5- C1

## World Bowling Championships

This is it... the finals at the *World Bowling Championships*. Only two competitors remain; Sven Tensplit and Gunther Ball. The winner of the match will be crowned the World Champion and win a significant cash prize along with a trophy commemorating the victory.

The event organizer needs help calculating the value of the cash prize. He has many expenses including the renovation of the bowling lane and purchasing the trophy. However he will make a profit by selling limited edition items like bowling balls, sports bags and sports bags as well as TV advertisements. One third of the profit earned will go towards the cash prize.

**You must determine who wins the tournament and how much prize money they earn.**

**You must take the following constraints into consideration:**

- The winner is the person who earns the most points in the match
- The revenue is generated by selling bowling balls, sports bags and showing TV Ads
- The costs include constructing the trophy and renovating the bowling lane, and
- Prize money is 1/3 of the profit the event organizer earns

### Match Winner

The following table shows the final score card indicating points for each round.

| Round   | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | Total Points |
|---------|----|----|----|----|----|----|----|----|----|----|--------------|
| Sven    | 30 | 30 | 30 | 28 | 20 | 20 | 30 | 20 | ?  | ?  | ?            |
| Gunther | 30 | 30 | 30 | 26 | 19 | 20 | ?  | 18 | 30 | 28 | ?            |

Sven's score for the 9<sup>th</sup> and 10<sup>th</sup> rounds are missing. An audience member remembers that the number of points earned in round 9 is the mean score from Sven's first 8 rounds. Sven's score from round 10 is the same as his score from round 9.

Gunther's score in the 7<sup>th</sup> round is the same as the median score he earned from the first 6 rounds.

### Bowling Balls

Limited edition World Bowling Championship bowling balls will be sold at the tournament. A total of 100 bowling balls will be sold.

- The surface area of the spherical bowling ball is 1256 cm<sup>2</sup>
- The ball is composed of an advanced polymer resin which costs \$0.0191 / cm<sup>3</sup> to manufacture
- Each ball will be sold for \$250

### Sports Bags

Limited edition *World Championship Bowling* sports bags will also be sold at the tournament. A total of 1000 large sports bags and 1000 small sports bags will be sold. The bags are in the shape of square based prisms. The large sports bag is similar to the small sports bag.

Large Sports Bag



Small Sports Bag



- The area of the square base of the large sports bag is 0.36 m<sup>2</sup>.
- The length of the large sports bag is 75 cm.
- The area of the square base of the small sports bag is 0.16 m<sup>2</sup>
- The revenue earned from selling a bag is based on the following formula:

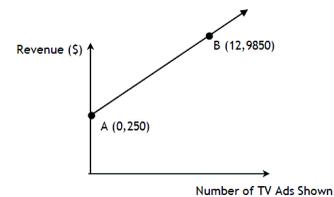
$$y = 12.5x \quad \text{where } x \text{ is the surface area of the bag (in m}^2\text{)} \\ y \text{ is the profit (in \$)}$$

### Television Advertisements

The tournament will be shown on two different channels; the Canadian Bowling Channel (CBC) and the Elderly Sports Network (ESN). Television advertisements, or TV Ads, broadcasted during the event will generate revenue for the tournament organizer.

The revenue earned from both channels follows partial linear relationships. The number of TV Ads and the revenue are the same from both channels this year.

Canadian Bowling Channel (CBC)



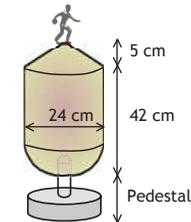
The Elderly Sports Network (ESN)

| Number of TV Ads Shown | Revenue (\$) |
|------------------------|--------------|
| 0                      | 700          |
| 5                      | 4550         |
| 12                     | 9940         |

The total revenue will be generated by the TV Ads being shown on **both** channels.

### The Gold Trophy

The surface of the trophy will be coated in real gold. The part that will be coated is composed of a cone, cylinder and hemisphere. The figurine on top and the wooden pedestal will not be coated.

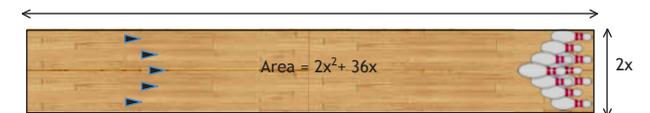


#### Properties of the Trophy

- The cone has a height of 5 cm
- The diameter of the cone, cylinder and sphere are all 24 cm.
- The total height of the cylinder and hemisphere is 42 cm.
- Gold costs \$4.65 / cm<sup>2</sup>
- The pedestal and figurine do not cost anything

### The Bowling Lane

The top view of a bowling lane is shown below. The whole surface needs to be re-done. The dimensions of the surface are given as polynomials. The total perimeter around the rectangular lane is 40.8 m. The cost to resurface the wood will be \$332.45 / m<sup>2</sup>.



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# Marking Key

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## Step 1 Determining Who Wins the Match

a) Sven's total points and missing scores for round 9 and 10

$$\begin{aligned} \text{mean} &= \frac{30 + 30 + 30 + 28 + 20 + 20 + 30 + 20}{8} \\ &= \frac{208}{8} \\ x &= 26 \end{aligned}$$

$$\text{Total point } s = 208 + 26 + 26 = 260$$

b) Gunther's total points and missing score for round 7

$$19 \ 20 \ 26 \ 30 \ 30 \ 30$$

$$\text{Median} = (26 + 30) / 2 = 28$$

$$\text{Total points} = 30 + 30 + 30 + 26 + 19 + 28 + 18 + 30 + 28 = 259$$

Therefore Sven wins by 1 point.

## Step 2 Tournament Revenues

### Limited Edition Bowling Balls

a) Determining the radius and the volume of the bowling ball

$$\begin{aligned} A_r &= 4\pi r^2 & V &= \frac{4}{3}\pi r^3 \\ 1256 &= 4\pi r^2 & V &= \frac{4}{3}\pi(10)^3 \\ r^2 &= \frac{1256}{4\pi} & V &= 4186.7 \text{ cm}^3 \\ r^2 &= 100 & & \\ r &= 10 \text{ cm} & & \end{aligned}$$

b) Determine the profit for selling the 100 bowling balls

$$4186.7 \text{ cm}^3 \times \$0.0191 / \text{cm}^3 = \$80$$

$$\text{Profit} = \text{sales price} - \text{cost} = \$250 - \$80 = \$170$$

## Step 2 Sports Bags

a) Determine the surface area of the large sports bag by finding the side length of the base since it is a square then using the surface area formula.

$$\begin{aligned} s &= \sqrt{A} \\ s &= \sqrt{0.36} = 0.6 \text{ m} \end{aligned}$$

$$\begin{aligned} A_r &= 2A_b + P_b h \\ A_r &= 2(0.36) + (4 \cdot 0.6)(0.75) \\ A_r &= 2.52 \text{ m}^2 \end{aligned}$$

c) Since the bags are similar, determine the surface area of the small sports bag by using the scale factor.

$$\begin{aligned} k^2 &= \frac{A_{\text{big}}}{A_{\text{small}}} = \frac{0.36}{0.16} = 2.25 \\ k &= \sqrt{1.44} = 1.5 \end{aligned}$$

$$\begin{aligned} h_{\text{small}} &= h_{\text{big}} / k \\ h_{\text{small}} &= 75 / 1.5 \\ &= 50 \text{ cm} \\ A_r &= 2A_b + P_b h \\ A_r &= 2(0.16) + (4 \cdot 0.4)(0.5) \\ A_r &= 1.12 \text{ m}^2 \end{aligned}$$

e) The profit earned by the large sports bag is

$$\begin{aligned} y &= 12.5x \\ y &= 12.5(2.52) \\ y &= \$31.50 \text{ for a big bag } \times 1000 \text{ is } \$31\ 500 \end{aligned}$$

$$\begin{aligned} y &= 12.5x \\ y &= 12.5(1.12) \\ y &= \$14 \text{ for a small bag } \times 1000 \text{ is } \$14\ 000 \end{aligned}$$

The total profit of the all the bags is \$45 500.

**Step 3 Television advertisements****a) Find the equation of the line for the Canadian Bowling Channel**

$$a = \frac{y_2 - y_1}{x_2 - x_1} \quad b = 250$$

$$a = \frac{9850 - 250}{12 - 0} \quad y = 800x + 250$$

$$a = 800$$

**b) Find the equation of the line for the Elderly Sports Network ESN**

$$a = \frac{y_2 - y_1}{x_2 - x_1} \quad b = 650$$

$$a = \frac{9940 - 4550}{12 - 5} \quad y = 770x + 700$$

$$a = \frac{5390}{7}$$

$$a = 770$$

**c) Find the point of intersection**

|                           |                     |                     |
|---------------------------|---------------------|---------------------|
| <i>let</i> $y_1 = y_2$    | <i>check</i>        | $y = 800x + 250$    |
| $800x + 250 = 770x + 700$ | $y = 770x + 700$    | $y = 800(15) + 250$ |
| $30x = 450$               | $y = 770(15) + 700$ | $y = 12\,250$       |
| $x = 15$                  | $y = 12\,250$       |                     |

Therefore the total money earned is  $2 \times \$12\,250$  which is  $\$24\,500$ **Step 4 Coating the Trophy in Gold****a) Find the slant length of the cone**

$$SL^2 = r^2 + h^2$$

$$SL^2 = 12^2 + 5^2$$

$$SL^2 = 144 + 25$$

$$SL^2 = 169$$

$$SL = 13 \text{ cm}$$

**b) Calculate the lateral surface area of the cone**

$$A_{\text{cone}}$$

$$A_L = \pi r SL$$

$$A_L = 3.14(12)(13)$$

$$A_L = 489.84 \text{ cm}^2$$

**c) Calculate the height of the cylinder**

$$h_{\text{cylinder}} = 42 \text{ cm} - r_{\text{hemisphere}} = 42 \text{ cm} - 12 \text{ cm} = 30 \text{ cm}$$

**d) Calculate the lateral surface area of the cylinder**

$$A_L = 2\pi r h$$

$$A_L = 2(3.14)(12)(30)$$

$$A_L = 2260.8 \text{ cm}^2$$

**e) Calculate the lateral surface area of the hemisphere**

$$A_{\text{cone}}$$

$$A_L = 2\pi r^2$$

$$A_L = 2(3.14)(12)^2$$

$$A_L = 904.32 \text{ cm}^2$$

**f) Sum up the total area and determine the cost of coating the trophy in gold**

$$A_T = 489.84 \text{ cm}^2 + 2260.8 \text{ cm}^2 + 904.32 \text{ cm}^2$$

$$A_T = 3654.96 \text{ cm}^2$$

$$\text{Cost} = \$4.65 / \text{cm}^2 \times 3654.96 \text{ cm}^2 = \text{approximately } \$17\,000$$

**Step 5 Refurbishing the Lanes**

a) Determine the length of the bowling alley

$$l = \frac{A}{w} = \frac{2x^2 + 36x}{2x}$$
$$l = x + 18 \text{ m}$$

b) Determine x given the perimeter

$$P = 2l + 2w$$
$$40.8 = 2(x + 18) + 2(2x)$$
$$40.8 = 6x + 36$$
$$4.8 = 6x$$
$$x = 0.8$$

c) Sub  $x=0.8$  back into find the area and the cost

$$A = 2x^2 + 36x$$
$$A = 2(0.8)^2 + 36(0.8)$$
$$A = 30.08 \text{ m}^2$$

$$\text{Cost} = \$332.45 / \text{m}^2 \times 30.08 \text{ m}^2$$
$$= \$10\,000$$

The total cost of refurbishing the lane is \$10 000

**Step 6 Total Cost of Everything**

|                       | Cost (\$) |
|-----------------------|-----------|
| Bowling Balls         | 17 000    |
| Small Sports Bags     | 14 000    |
| Large Sports Bags     | 31 500    |
| TV Advertisements     | 24 500    |
| Gold Trophy           | -17 000   |
| Refurbishing the Lane | -10 000   |
| Total Profit          | 60 000    |

A third of \$60 000 is \$20 000.

\*Total revenues is \$87 000 and total costs are \$27 000\*

**Answer: Sven wins the WBC and earns \$20 000 as a cash prize.**