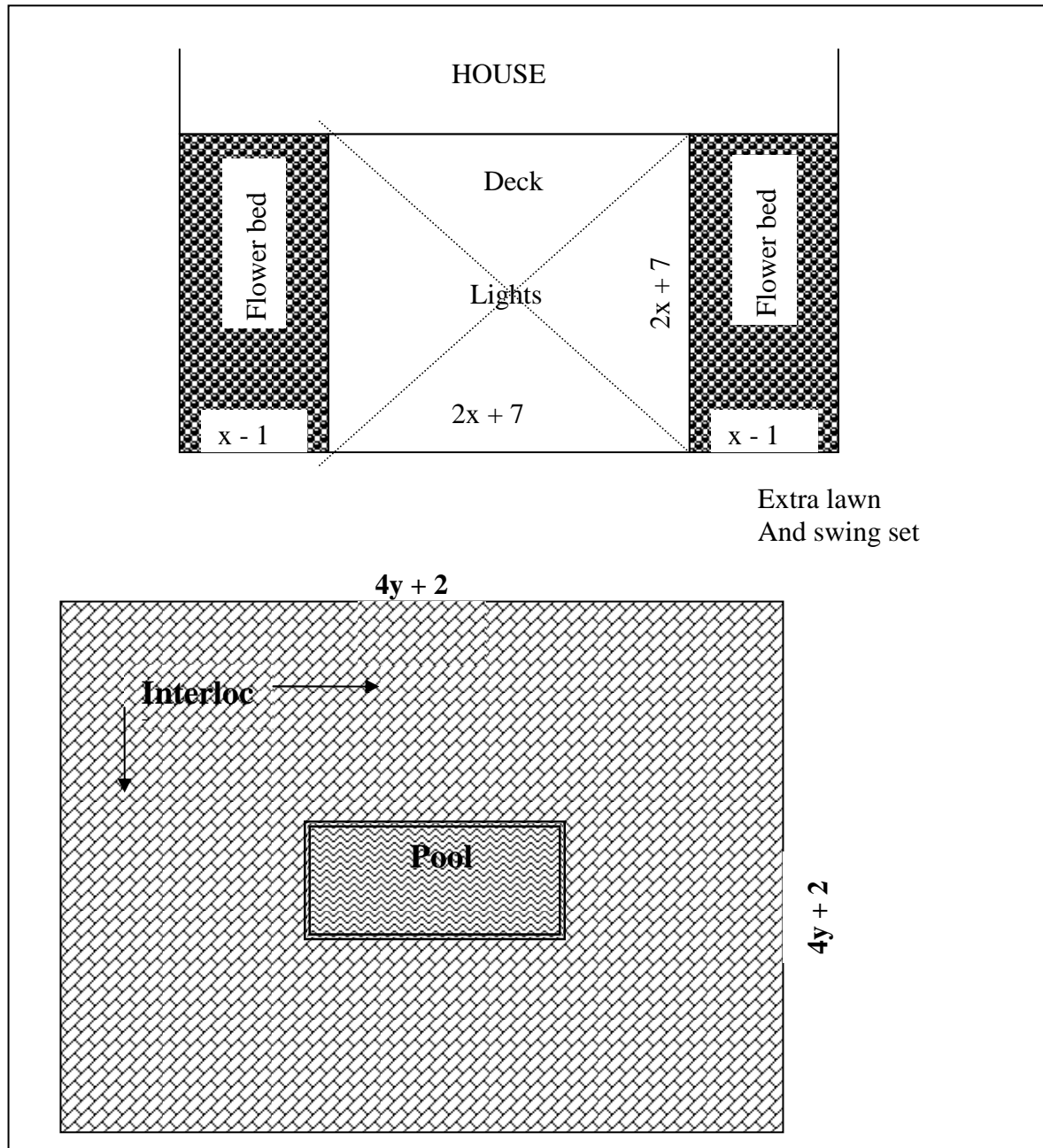


Mr. Lee's Backyard Design

Mr Lee's family has been setting money aside every year for landscaping their backyard. Now that they have saved up \$ 40 000, they think they are ready to do it this summer.

They have a large lot with just a patio and a swing set for the kids. They would like to have a raised deck instead, a couple of flower beds for Mrs. Lee to maintain, an in-ground swimming pool with interlock around it for lounging on the pool. And for the safety of his children he will have that section fully fenced. After discussing the potential of the available space, they came up with the following design.



Notes:

- 1) The deck is square with side length $(2x + 7)$; $x \in \mathbb{R}$
- 2) Lights will be hung above the deck diagonally from each corner.
- 3) The 2 flower beds are identical
- 4) The total width of the deck and flower beds must be the same as the width of the house which is 13 m.
- 5) The fenced section of interlock and pool is a square, but the pool is a rectangle
- 6) The section that will be covered with interlock has an area of $(12y^2 + 16y + 8)$; $y \in \mathbb{Z}$
- 7) The surface area that will be dug out for the pool must be more than 30 m^2 and less than 35 m^2 , for the flat price he was given.

After shopping around and researching prices Mr Lee organized his findings in the following table: (Prices **do not yet** include the 13% tax)

Item	Price
Deck – installed	\$100 / m^2
Dirt for flower beds	\$15 / m^2
Pool with S.A between 30 and 35 m^2	\$10 000
Interlock	\$50 / m^2
Fence	\$75 / m
String of lights	\$40 / 8m

Can Mr. Lee afford to landscape this year?

_____ Yes he can with \$ _____ remaining.

_____ No he can not, he will be short \$ _____.

Solution to Mr Lee's backyard

Deck:

$$\begin{aligned}2(x-1) + 2x + 7 &= 13 \\2x - 2 + 2x + 7 &= 13 \\4x + 5 &= 13 \\4x &= 8 \\x &= 2\end{aligned}$$

The deck is $(2x + 7)$ m by $(2x + 7)$ m ;
or 11m by 11 m

Area of deck is $(11)(11) = 121 \text{ m}^2$

Cost to install the deck = $(121)(100) = \$ 12\ 100$

Lights:

$$\begin{aligned}c^2 &= a^2 + b^2 \\&= 11^2 + 11^2 \\&= 242 \\c &= 15.5\end{aligned}$$

2 strings = $2(15.5) = 31 \text{ m}$
Number of packages needed: $31/8 = 3.8$
So we need 4 packages.

Cost of lights = $4(40) = \$ 160$

Flower Bed: dimensions : $(x - 1)$ by $(2x + 7)$
That is : $(2 - 1)$ by $(4 + 7)$
Or : 1 m by 11 m
2 flower beds have total area of : $2(11) = 22 \text{ m}^2$

Cost of dirt = $(22)(15) = \$ 330$

Pool:

Area of pool = total area – Area of interlock

$$\begin{aligned}&= (4y + 2)^2 - (12y^2 + 16y + 8) \\&= 16y^2 + 16y + 4 - 12y^2 - 16y - 8 \\&= 4y^2 - 4\end{aligned}$$

$$\begin{aligned}30 &\leq \text{Area of pool} \leq 35 \\30 &\leq 4y^2 - 4 \leq 35 \\34 &\leq 4y^2 \leq 39 \\8.5 &\leq y^2 \leq 9.75 \\2.92 &\leq y \leq 3.12\end{aligned}$$

Since $y \in \mathbb{Z}$; then $y = 3$
So the pool is 8 m by 4 m with area of 32 m^2

Cost of pool is fixed = $\$ 10\ 000$

Interlock:

$$\begin{aligned}y = 3 &= 12y^2 + 16y + 8 \\&= 12(3)^2 + 16(3) + 8 \\&= 108 + 48 + 8 \\&= 164 \text{ m}^2\end{aligned}$$

Fence:

$$\begin{aligned}y = 3 &= 4(4y + 2) \\&= 4(4(3) + 2) \\&= 4(14) \\&= 56 \text{ m}\end{aligned}$$

Cost of interlock = $(164)(50) = \$ 8\ 200$

Cost of fence = $(56)(75) = \$ 4\ 200$

Sub total cost = $12\ 100 + 160 + 330 + 10\ 000 + 8\ 200 + 4\ 200$
= $\$ 34\ 990$

Taxes = $34\ 990(0.13) = \$ 4\ 548.70$

Total cost = $34\ 990 + 4\ 548.70 = \$ 39\ 538.70$

Therefore, yes he can with $\$ 461.30$ remaining