Situational Problem- The vegetable garden

Your uncle has asked for your help to determine the cost to purchase and install a fence around his vegetable gardens, in order to keep out his dog, rabbits and squirrels.

He plans to have 3 different sections, one of each for tomatoes, cucumbers, and strawberries. You need to find the total length of fence needed and calculate the cost to make one purchase and one installation.

While waiting for him to provide you with the dimensions, you start researching the costs.

Purchase cost:

The cost of purchasing the fence depends on its length. In an attempt to find the linear function that associates the cost and length, you ask a few neighbours how much they paid for their fences. You collect the following information:

Fence costs

Length of fence (m)	Cost ($)
21	323
45	635
50	700

The total length of the fence will be rounded to the nearest meter.

Installation cost:

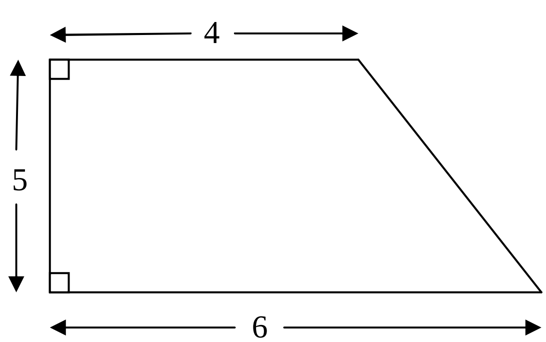
You phone two companies to determine which one is cheaper to install the fence. Their rates are as follows:

Awesome fences rates: $f(x) = 10.25x + 50$
Best fences rates: $f(x) = 9.75x + 90$

Where x is the total length of the fence in meters; and f(x) is the cost in dollars.

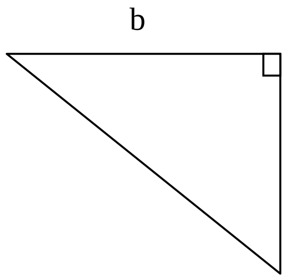
Your uncle then provides you with the following information about each section. (Note: the pictures are not to scale. All measurements are in meters)

1) Tomatoes:



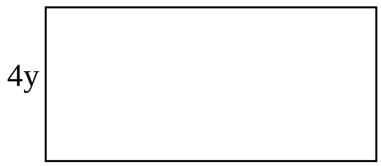
This section is in the shape of a right trapezoid with the given dimensions.

2) Cucumbers:



This section is in the shape of a right triangle. The height is 10 less than twice the base. Your uncle notices that if he increases the height by 4, the area increases by 16. This gives him more space to grow enough cucumbers to pickle. Since he has the necessary space, he decides to go with the bigger dimensions.

3) Strawberries:



This section is in the shape of a rectangle. The width is 4y, and the maximum area is $12y^2 + 4y$. The length is at most 7; and $y \in \mathbb{R}$.
Your uncle will decide later what exact dimensions to use for this section.

After doing all the calculations you must prepare a summary for your uncle with your results.

Your Summary for your uncle:

If the total fence length is __________m; then both Awesome fences and Best fences will charge the same cost of $_____________for installation.

Maximum possible total fence length is ________________ m ;

It will cost $ _______________ to purchase.

It will cost $ _______________ to install by company ___________________

Maximum total cost for your uncle to purchase and install the fence is $ _______________.

Solution

Purchase cost

$$ROC = a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{635 - 323}{45 - 21} = \frac{312}{24} = 13$$

$b = y - ax = 323 - (13)(21) = 50$

Linear function: $y = 13x + 50$

Installation cost

Solve the system:

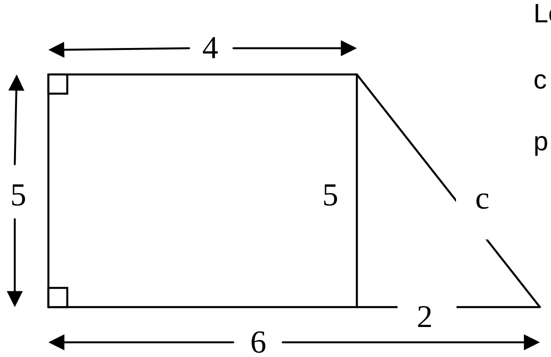
$$10.25x + 50 = 9.75x + 90$$
$$10.25x = 9.75x + 40$$
$$0.50x = 40$$
$$x = 80$$

$y = 10.25x + 50$	$y = 9.75x + 90$
$y = 10.25(80) + 50$	$y = 9.75(80) + 90$
$y = 870$	$y = 870$

Solution: (80, 870)

Awesome fences is a better choice for lengths below 80 meters, while Best fences is the better choice for fences longer than 80 meters.

Section 1: Tomatoes (Trapezoid)



Leg = 6 – 4 = 2

$c = \sqrt{5^2 + 2^2} = \sqrt{25 + 4} = \sqrt{29} \approx \underline{5.385\text{ m}}$

$p \approx 5.385 + 4 + 5 + 6 \approx 20.385$ m

Section 2: Cucumbers (Right triangle)

base = b   Height = 2b – 10   New dimensions: base = b   height = 2b – 10 + 4 = 2b – 6

Area = $\frac{b \times h}{2}$:

$$\frac{(b)(2b - 10)}{2} + 16 = \frac{(b)(2b - 6)}{2}$$
$$\frac{2b^2 - 10b}{2} + 16 = \frac{2b^2 - 6b}{2}$$
$$b^2 - 5b + 16 = b^2 - 3b$$
$$-5b + 16 = -3b$$
$$16 = 2b$$
$$8 = b$$

Height = 2b – 6 = 2(8) – 6 = 10 m

Hypotenuse $c = \sqrt{8^2 + 10^2} = \sqrt{64 + 100} = \sqrt{164} \approx$ 12.806 m

$p \approx 12.806 + 8 + 10 \approx$ 30.806 m

Section 3: Strawberries (Rectangle)

Area = length x width

Calculations using the maximum length of 7 m

$12y^2 + 4y \leq 7 \times 4y$

$12y^2 + 4y \leq 28y$

$12y^2 \leq 24y$

$y \leq 2$

$p = 4y + 7 + 4y + 7 = 4(2) + 7 + 4(2) + 7 =$ 30 m

Maximum perimeter for all 3 sections: $p \approx 20.385 + 30.806 + 30 \approx 81.191$ m

Purchase cost: $y = 13x + 50 \approx 13(81.191) + 50 \approx$ \$1 105.49

Installation cost:

Awesome fences: $f(x) = 10.25x + 50 \approx 10.25(81.191) + 50 \approx$ \$882.21

Best fences: $f(x) = 9.75x + 90 \approx 9.75(81.191) + 90 \approx$ \$881.62

After doing all the calculations you must prepare a summary for your uncle with your results.

Your Summary for your uncle:

If the total fence length is ____**80____ m**; then both Awesome fences and Best fences will charge the same cost of **\$_____870______** for installation.

Maximum possible total fence length is ____**≈ 81.191 _____m**;

It will cost **\$____1 105.49______** to purchase.

It will cost **\$_____881.62_______** to install by company __**Best fences___**

Maximum total cost for your uncle to purchase and install the fence is **\$_____1 987.11_____**.